SMITHSONIAN INSTITUTION
ASTROPHYSICAL OBSERVATORY

Research in Space Science

SPECIAL REPORT

Number 230

PROBABILITY OF RECORDING SATELLITE IMAGES OPTICALLY

Kurt Lambeck

December 5, 1966

CAMBRIDGE, MASSACHUSETTS, 02138

SAO Special Report No. 230

PROBABILITY OF RECORDING SATELLITE IMAGES OPTICALLY

Kurt Lambeck

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts, 02138

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

BIOGRAPHICAL NOTE

Mr. Lambeck graduated in geodesy from the University of New South Wales, Australia, in 1963. He studied at the Geodetic Institute, Delft, Holland, in 1964 and at the National Technical University of Athens in 1965. He is currently working at the Department of Surveying and Geodesy, Oxford University, England, and was Consultant Geodesist with the Smithsonian Astrophysical Observatory during the summer of 1966.

His interests lie in satellite geodesy, particularly in combining the results of different methods and of different observations.

ABSTRACT

A study is made of the relationship between satellite-tracking cameras and objects now in orbit in order to establish a simple criterion for predicting the frequency with which any particular satellite can be observed with a specific camera. A comparison of different cameras, based on efficiency, is then possible.

PROBABILITY OF RECORDING SATELLITE IMAGES OPTICALLY

Kurt Lambeck

The purpose of this note is to give certain criteria that will assist in predicting the answers to questions such as the following: What satellites can be tracked with a particular camera? And, if a satellite can be observed at all, what will be the frequency or probability with which such observations can be made? Or, in order to track optically a certain object, what camera characteristics are desired for optimum tracking efficiency?

Generally, only the relative merits of various camera-satellite combinations will be of consequence, so that several simplifying assumptions can be made. Foremost are that the satellite is assumed to pass over all parts of the station-coverage area with equal frequency, and that the sky brightness is not considered. Further, only satellites in circular orbits are treated and the earth's rotation and precession of the orbital plane have been neglected. A more detailed account, taking into consideration the inclination of the object, the latitude of the station, and sky brightness, has been presented elsewhere (Lambeck, 1966).

The satellite's velocity relative to the observer is a function of the satellite height above the earth, h, the zenith distance, z, the satellite range, r, and the direction in which the object is moving relative to the observer. Denoting the object's angular velocity relative to the center of the earth by $\omega_{C.E.}$, the maximum apparent angular velocity, ω_{max}, will be

$$\omega_{max} = \omega_{C.E.} \frac{R + h}{r} \text{ radians/sec} \quad ,$$

This work was supported in part by grant NsG 87-60 from the National Aeronautics and Space Administration.

and the minimum apparent velocity will be

$$\omega_{min} = \omega_{C.E} \frac{R + h}{r} \cos (z - \eta) \quad ,$$

R being the earth's radius and η the subsatellite distance, which is related to satellite height and zenith distance by the expression

$$\sin (z - \eta) = \frac{R}{R + h} \sin z \quad .$$

Intermediate velocities are a function of the direction in which the satellite is moving, but in order for the number of parameters to be kept to a minimum it will be preferable to introduce a mean approximate velocity defined by

$$\omega_m = \sqrt{\frac{1}{2} \left(\omega^2_{max} + \omega^2_{min}\right)}$$

$$= \frac{R + h}{r} \omega_{C.E} \sqrt{\frac{1}{2} \left[1 + \cos^2 (z - \eta)\right]} \quad . \tag{1}$$

The magnitude, m, of a satellite is a function of its physical characteristics, such as its shape, size, and albedo, as well as its distance from the observer, while the photographic magnitude is also a function of the angular velocity of the object.

In the case of a spherical reflecting object, Zirker, Whipple, and Davis (1958) give

$$m = -14.13 - 2.50 \log k \frac{ab^2}{4r^2} E_0 \quad , \tag{2}$$

where a is the albedo, b the radius of the object, E_0 the intensity of the incident illumination on the satellite, and k the coefficient of atmospheric extinction. An expression similar to (2) exists for diffuse reflecting objects.

For any particular satellite, m will therefore be a function of z and r, or any two similar parameters.

The tracking power, P, of a camera is defined by

$$P = m_{lim} + 2.5 \log_{10} \omega \quad ,$$

m_{lim} being the limiting magnitude of the stars that are recorded while tracking with an angular velocity ω. Figure 1 illustrates the relationship among P, m, and ω for a nontracking camera. A similar quantity, Q, but one that is a function of the satellite characteristics, can be defined as

$$Q = m + 2.5 \log \omega$$

and will be referred to as the "tracking capacity" of the satellite. The value m will be given by expressions such as equation (2), and for ω expression (1) will be used.

The satellite is observable by the camera when

$$Q \geq P \quad . \tag{3}$$

Both Q and P are dependent on the zenith distance and range, but the functional relationships do differ so that condition (3) gives no information concerning the probability of Q exceeding P, or alternatively about the frequency with which the satellite can be observed.

Figure 2 illustrates the variations in magnitude, Δm, and in angular velocity as a function of z and h. Atmospheric extinction has been taken into account. The total magnitude will depend on the satellite's physical characteristics and on its height.

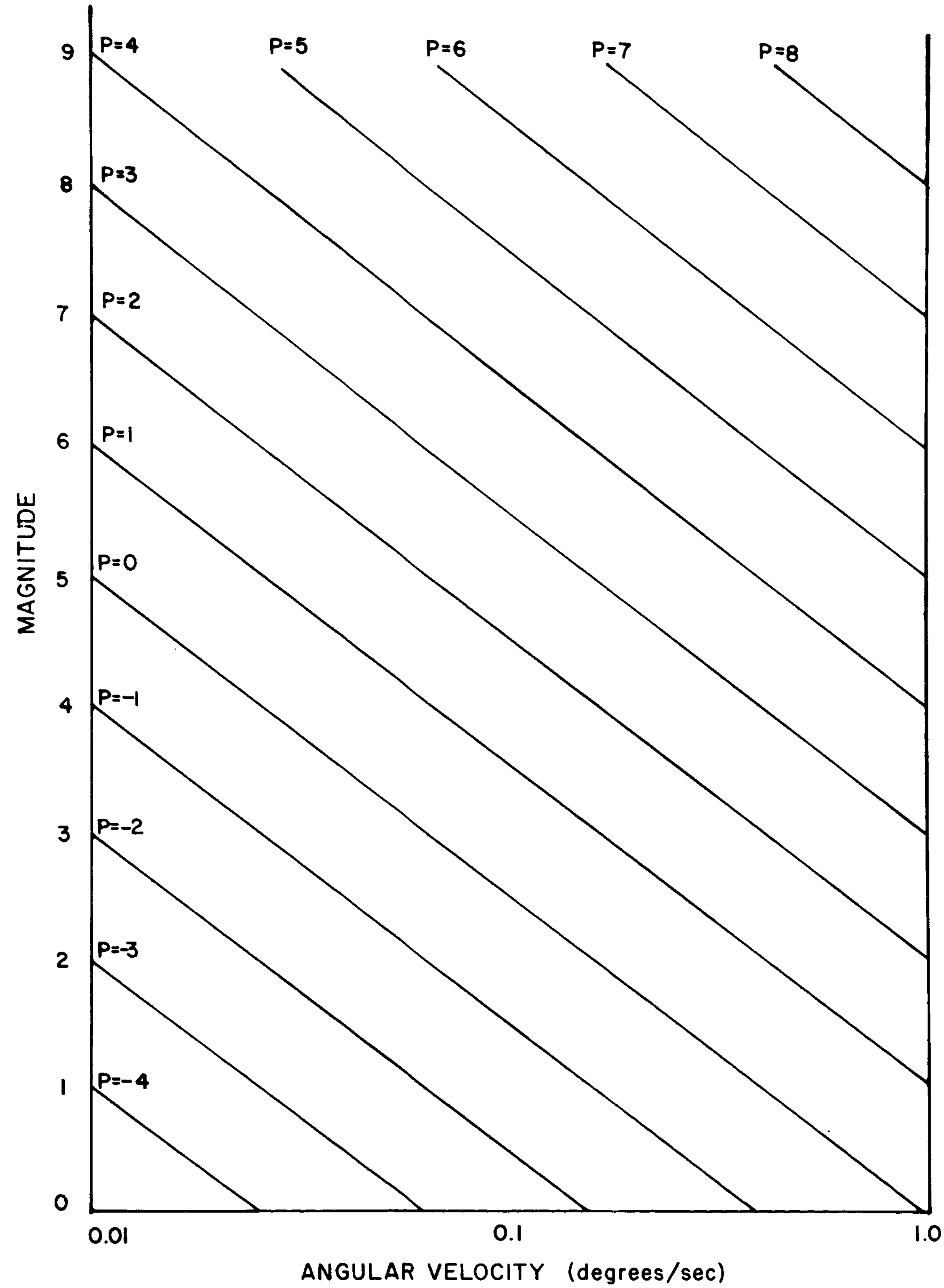

Figure 1. Relation between tracking power, magnitude, and angular velocity.

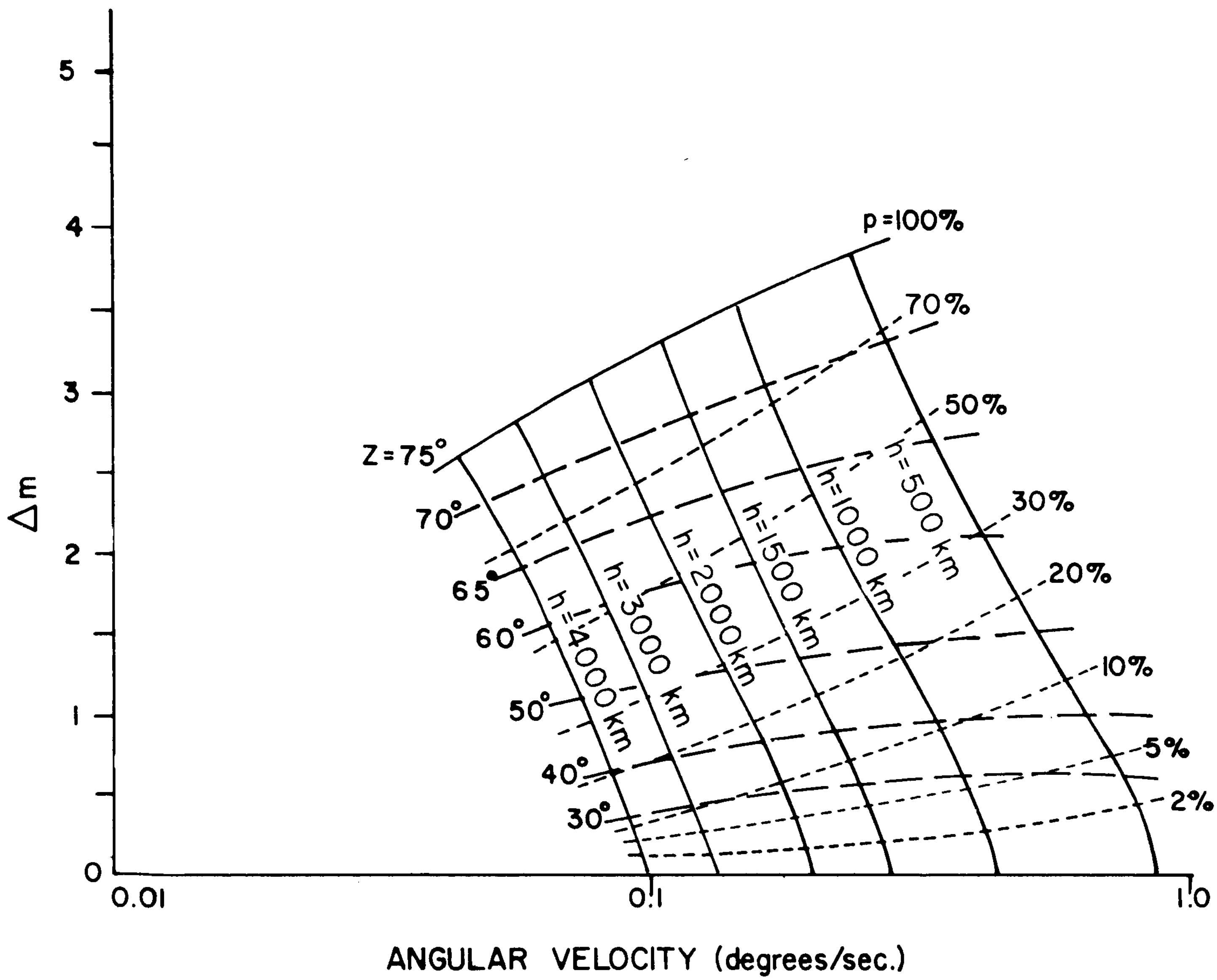

Figure 2. Nomogram giving the angular velocity and loss of magnitude, as a function of the height and zenith angle, and associated total possibilities.

If the relative probability of the satellite being above a 15° elevation is defined as 100% (SAO satellite predictions are for positions at least 15° above the horizon), the probability, p_i, of the object being at a zenith distance of less than z_i is, in view of the earlier stated assumptions, simply the ratio of the station-coverage areas corresponding to $z = z_i$ and $z = 75°$ for the height of the satellite considered. Thus,

$$p_i = \frac{(1 - \cos \eta_i)}{(1 - \cos \eta_{75})} ,$$

where η_i and η_{75} are the subsatellite distances corresponding to z_i and z_{75} and the height of the particular object.

To every z_i and h combination, there exists then a number, p_i, that specifies the relative probability of a satellite of height h being at a zenith distance of less than z_i.

Curves of equal probability have been superimposed on Figure 2 (broken lines), as have curves of equal zenith distance (dotted lines). Figures 1 and 2 provide the necessary information to determine the relative probabilities with which certain satellites may be observed using specific cameras.

Consider, for example, a satellite whose height is 700 km and whose stellar magnitude at the zenith is +4. From Figure 2, the curve expressing the variations of magnitude and angular velocity with z is obtained by interpolating for h = 700. When this curve is superimposed upon Figure 1 by the transformation of the origin of Figure 2 to the point m = +4 on Figure 1, the curve expresses the "absolute" magnitude of the object as a function of z, as well as giving the probabilities of the satellite having a zenith distance less than a specific value. These curves will be called the "tracking-capacity curves."

The intersection of these Q curves with a specific tracking-power value indicates the point at which Q = P. Above this point, the satellites can never be observed by a camera with this particular P value. By interpolation, the probability and the zenith distance corresponding to the intersection of the Q and P curves is obtainable. Thus, for the above satellite and for a tracking power of +4, the satellite can be observed only when it has a zenith distance of less than 20° and the relative probability is about 3%. If the tracking power is increased to +5, $z = 62°$ and $p = 40\%$. A tracking power of +6 yields $z = 75°$ and $p = 100\%$.

Figures 3, 4, and 5 give the tracking-capacity curves for various objects in orbit around the earth. Their relevant characteristics are tabulated in Table 1. Where the objects are in noncircular orbits, a mean height is used.

In the case of tracking cameras, the tracking power is simply the limiting magnitude of the stars that can be recorded by the camera's optics — emulsion properties when tracking with 1°/sec. Similarly, the tracking capacity for any particular satellite is merely its magnitude, and its variations are simply due to the increasing atmospheric extinction with increasing zenith distance.

Figure 6 gives those relationships with the probabilities as defined previously. Figure 7 is simply the two axes representing magnitude and probability scales.

For any particular satellite whose stellar magnitude (m_0) at the zenith is known, the "absolute" magnitudes as a function of z are obtained by transforming the origin of Figure 6 to the point corresponding to the m_0 value on the magnitude axis of Figure 7, and interpolating for satellite height.

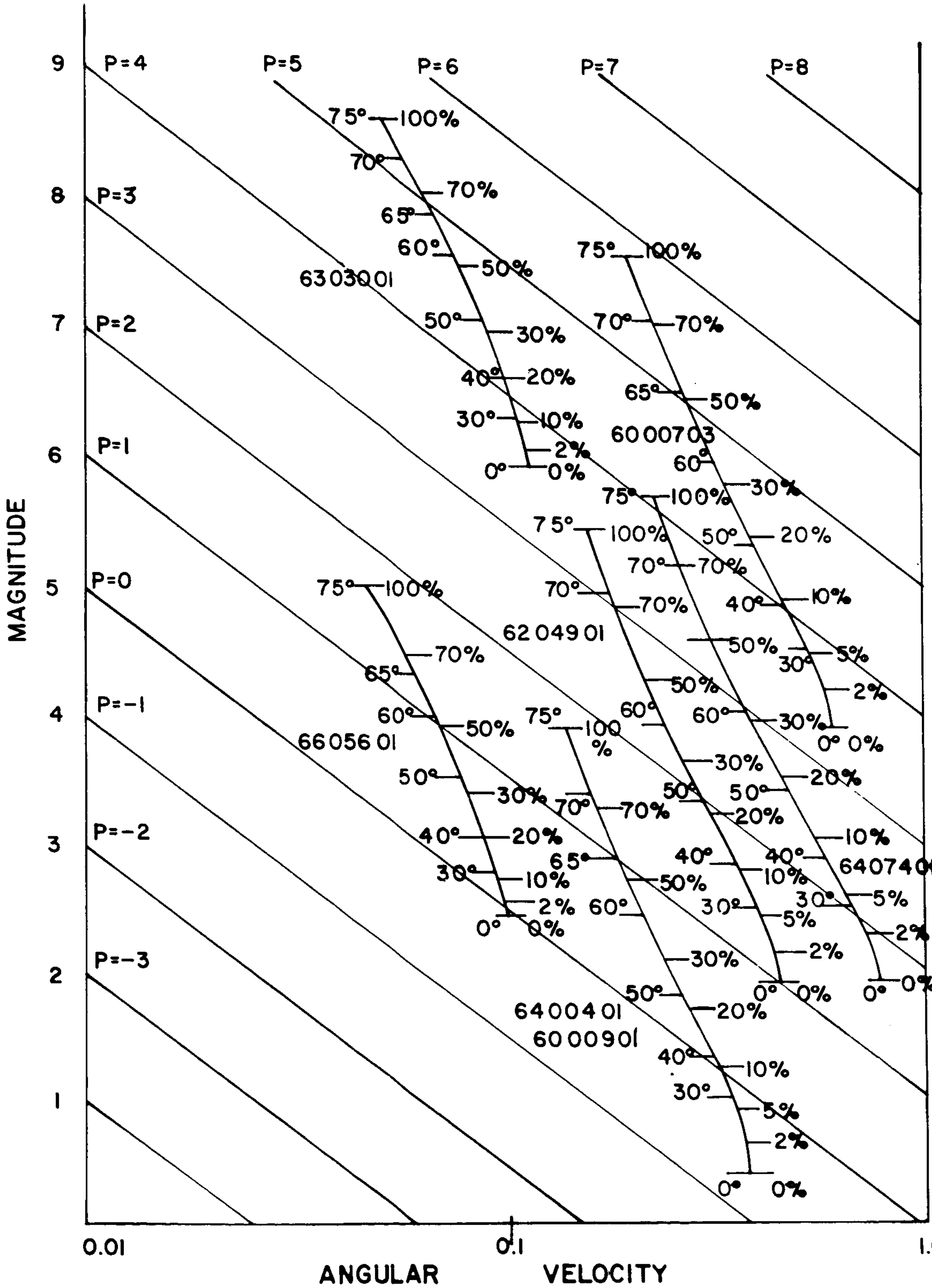

Figure 3. The tracking capacity curves for various satellites in orbit, associated total probabilities, and zenith distances.

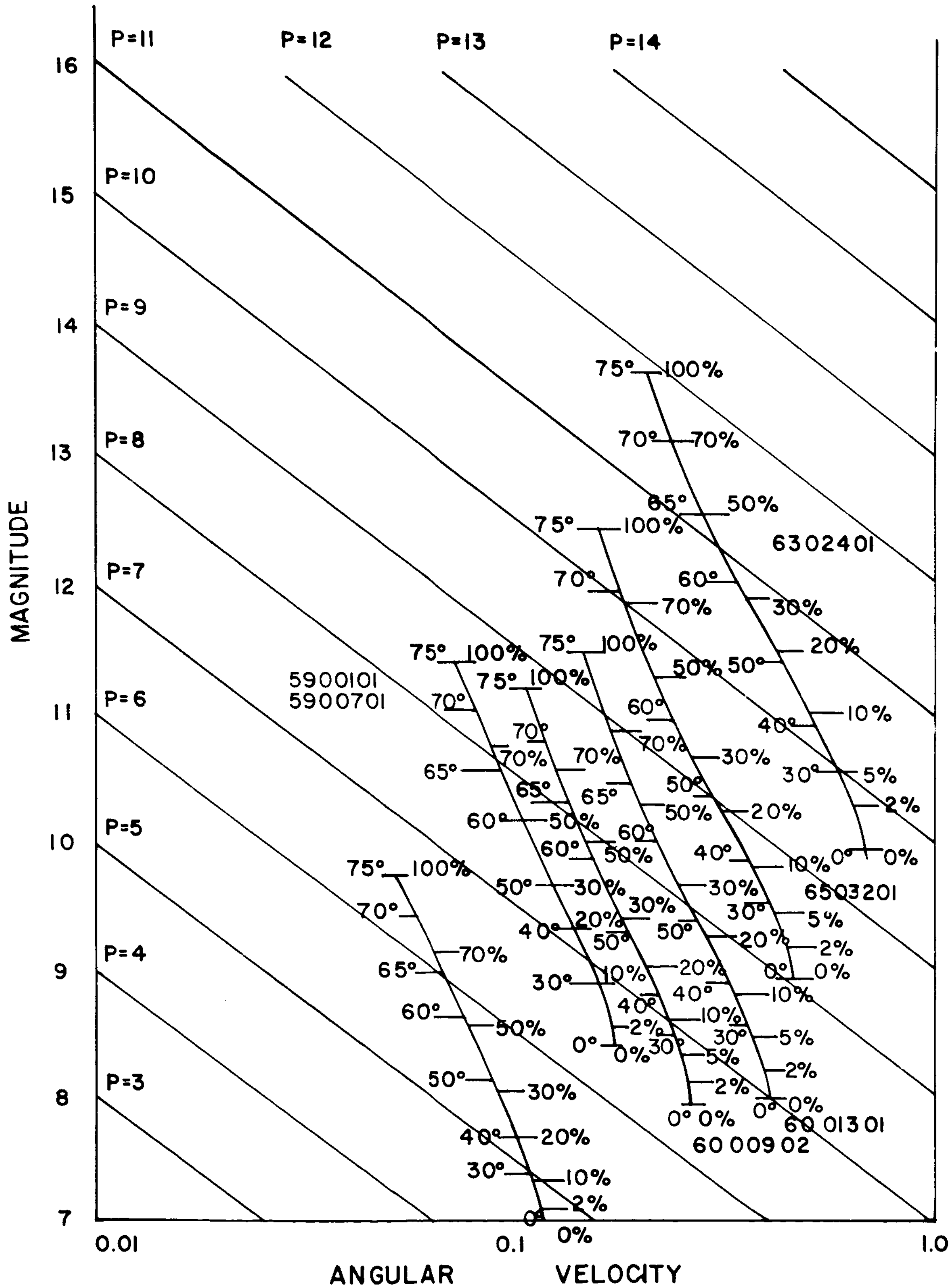

Figure 4. The tracking capacity curves for various satellites in orbit, associated total probabilities, and zenith distances.

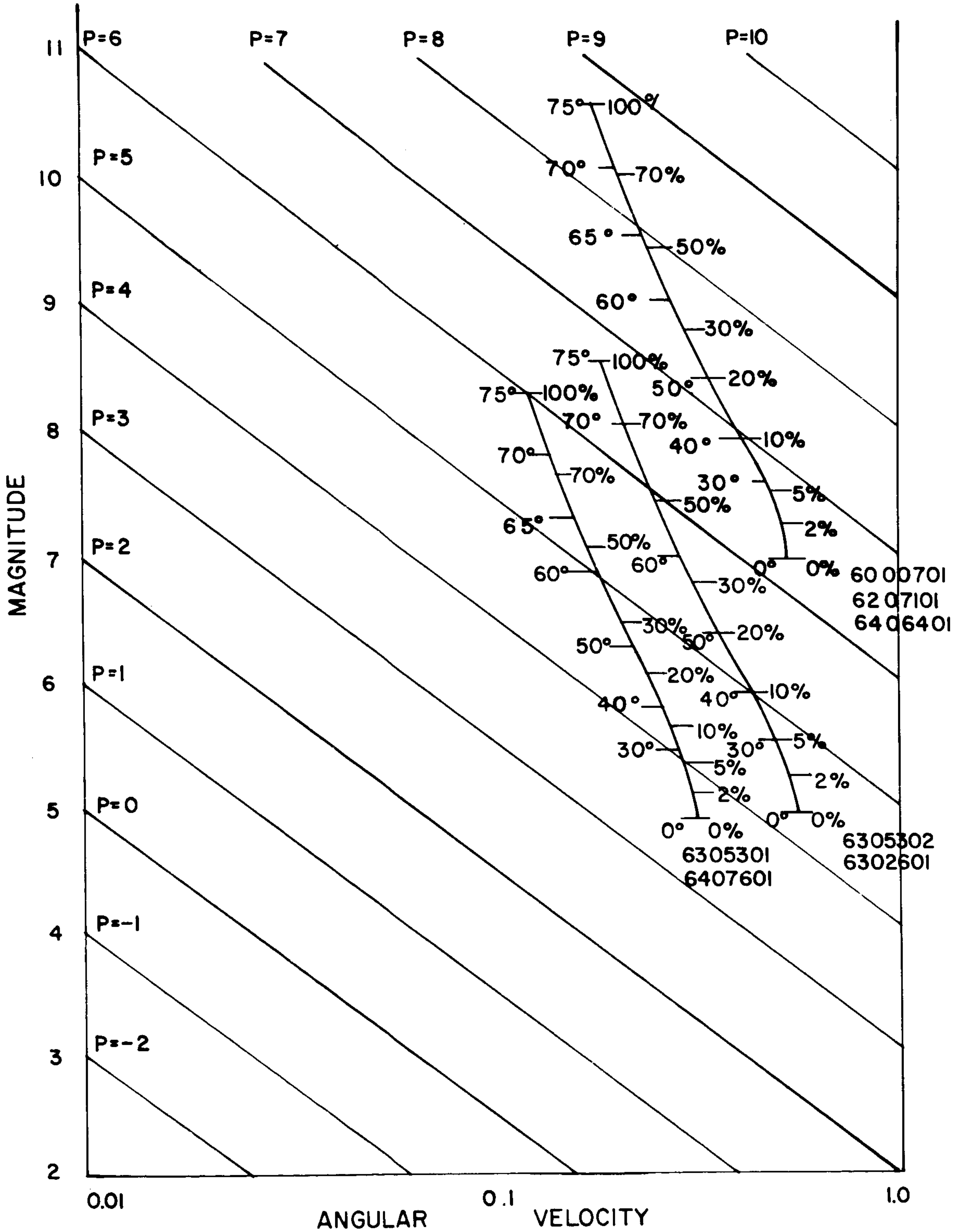

Figure 5. The tracking capacity curves for various satellites in orbit, associated total probabilities, and zenith distances.

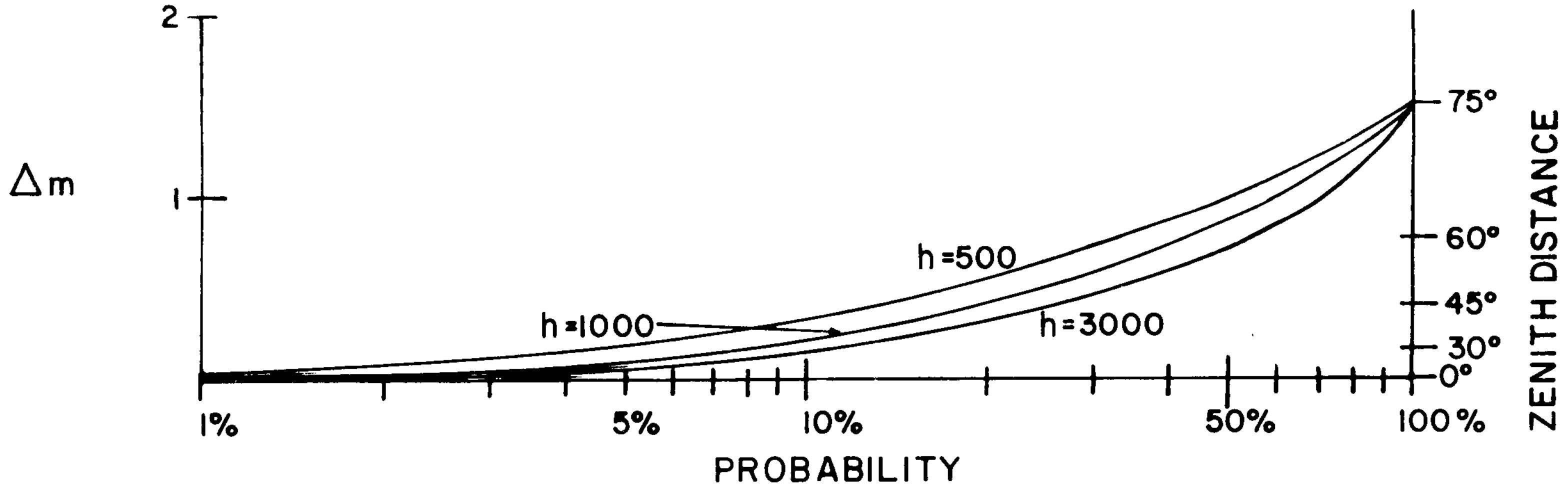

Figure 6. Total probability of a zenith distance below a certain value as a function of height.

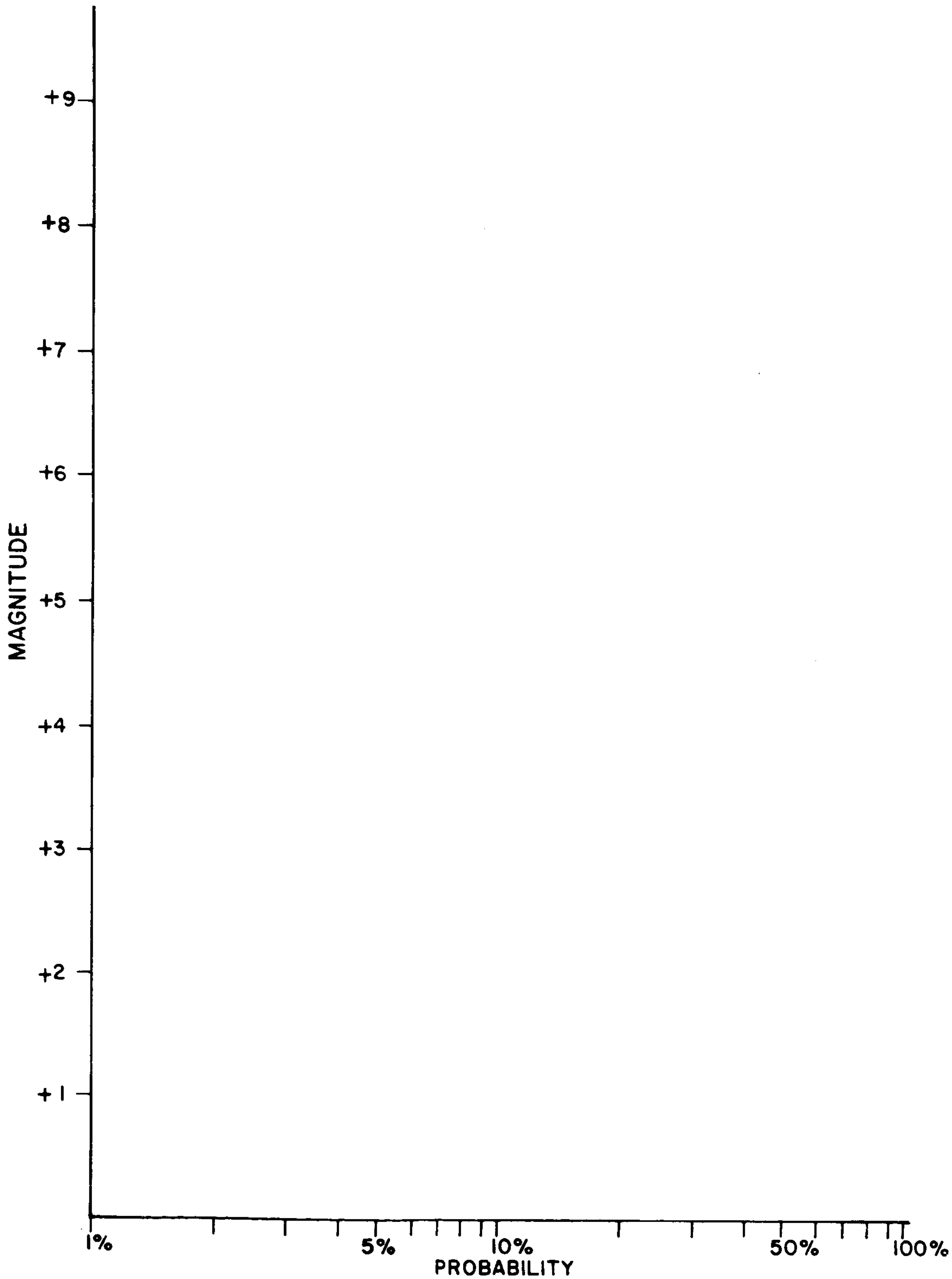

Figure 7. The axis with the magnitude and probability scales.

Table 1. Elevations and magnitudes of satellites considered in this study.

Satellite	H_{max}	H_{min}	m	Shape
59 001 01, Vanguard 1	3900	650	+8	Cylinder
59 007 01, Vanguard 2	3700	510	9	Cylinder
60 007 01, Transit 2A	1050	620	7	Sphere
60 007 03	1050	620	4	Cylinder
60 009 01, Echo 1	1600	1100	1	Sphere
60 009 02, Echo Rocket	1600	1500	8	Cylinder
60 013 01, Courier 1B	1200	1000	8	Sphere
61 028 01, Midas 4	3500		7	Cylinder
62 049 01	1000		2	Sphere
62 071 01	700		7	Cylinder
63 024 01, Tiros 7	621		10	Cylinder
63 026 01, Geophysical Satellite	1290	410	5	Cylinder
63 030 01	3700		6	Cylinder
63 053 01, Explorer 19	2250	700	5	Sphere
64 004 01, Echo 2	1200	1000	0	Sphere
64 053 02, Cosmos 44	800	700	5	Cylinder
64 064 01, Explorer 22	1100	900	7	Octagon
64 074 01, Explorer 23	980	500	2	Cylinder
64 076 01, Explorer 24	2400	600	5	Sphere
65 032 01, Explorer 27	1300	950	9	Cylinder
66 056 01, Pageos 1	4500	4000	2.5	Sphere

Thus, consider again the satellite of h = 700 km and m = +4. For P = +4, the satellite is obviously observable only when it is in the zenith, for P = +5, z = 65°, and p = 50%; while for P = +6, z = 75°, and p = 100%.

Figure 8 gives the magnitude, zenith distance, and probability relationships for the satellites tabulated in Table 1.

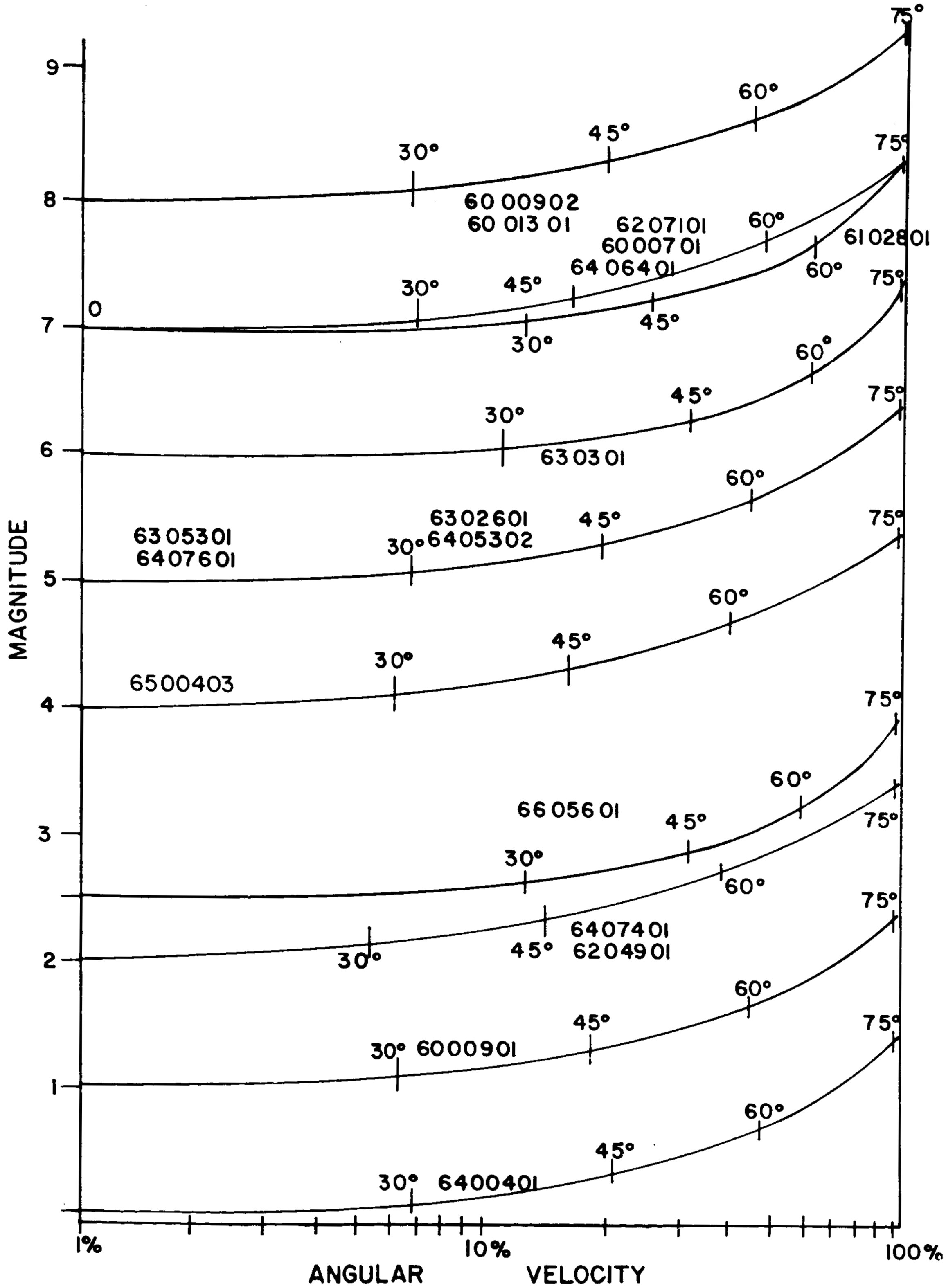

Figure 8. The total probabilities of magnitudes and zenith distances for some satellites.

REFERENCES

LAMBECK, K.

1966. Probability of recording satellite images optically. In The Use of Artificial Earth Satellites for Geodesy II, ed. by G. Veis (in press).

ZIRKER, J. B., WHIPPLE, F. L., DAVIS, R. J.

1958. Time available for the optical observation of an earth satellite. In Scientific Uses of Earth Satellites, ed. by J. A. Van Allen, Univ. of Michigan, Ann Arbor, Michigan, p. 29.